Introduction:

Weather has always been one of the most important natural phenomena that have had a profound impact on the day to day activities of human beings. Whether it has been a rainless summer that has affected crops or that it hasn't stopped raining for weeks on end, weather plays an important role in every person's life. However, over the centuries people have made great strides in being able to predict the weather with a lot of this improvement, at least in the scientific view, has been made over the 20th century. Some of these developments have included the use of radar, weather satellites, the evolution of dynamic meteorology, and the introduction of chaos into meteorology. But even with all of these advances some aspects of being able to predict what will happen in the coming days, or weeks still eludes meteorologists.

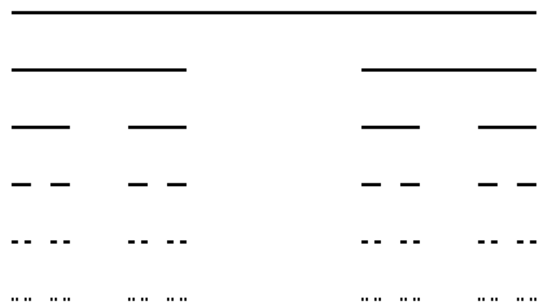
As Patrick Young once said, "The trouble with weather forecasting is that it's right too often for us to ignore it and wrong too often for us to rely on it." This is just as true today as it was the day that Patrick Young said it. Everyday weather forecasts are made trying to predict what will happen in the next couple hours, days or even weeks; but these forecasts are not always accurate because of how much chaos there is in the atmosphere. By the nature of chaos, we know that being able to predict the weather too far into the future is always going to be a challenge. But what about events that are the next couple of hours or days or maybe even weeks, shouldn't our present knowledge of the atmosphere and the capabilities of our instruments be able to give everyone really accurate and dependable weather forecasts? One of the main reasons that this is not always possible is because not every weather station has the capabilities to obtain fine data sets that will be able to help better predict the events that will unfold in the coming hours, days or weeks. The goal of this paper is to look into how fine data sets can be obtained from course data sets through the techniques of fractal analysis. This will be done by first looking into fractals, chaos and how these ideas have already been applied to meteorology, then course data sets and fine data sets will be defined to show exactly what is being looked at by this paper, then finally will outline how fractal analysis can be used to analyze the course data sets.

Research and Background Information:

Fractals have had a long history of being considered mainly mathematical oddities that have no practical application; however in recent years there has been a major shift to looking at fractals as a new kind of mathematics that can be used to describe numerous processes and objects in the natural world. The first time the term fractal was used in Benoit Mandelbrot's essay Fractals: Form, Chance and Dimension which was first published in 1975. Mandelbrot also gave the world a very simple definition of what a fractal is: "A fractal is a shape made of parts similar to the whole in some way". This definition was actually the second definition that he had used to define fractals, his first had not accounted for all things that can be considered fractals. This definition, although very vague, gives actually one of the simplest and easiest to understand definitions of what a fractal is and is what fractals will be defined as for the purposes of this paper.

Another aspect that helps mathematicians to recognize and investigate fractals is the idea that fractals can have non-integer dimensions. Although this is a very tough idea, even for many mathematicians, to wrap our heads around; it will make more sense once we look a couple of fractals.

The first and most simple fractal that most textbooks consider is called the Cantor set. This set is created by taking a line with a finite length and removing the middle third of the line. Then the middle third is removed from the two remaining thirds from the original line. The middle third is then removed from the four lines and this is repeated by removing the middle third from each of the remaining lines an infinite number of times. It leaves us with a series that looks like this:



If we were to zoom in on the pair of lines in the bottom row that are in the box we would find that it looked exactly like what is found in each of the columns of the fourth row. If we zoomed in again we would find that it still looked like what was in the fourth row. This could be done an infinite number of times, but the series would still look the same. So the Cantor Set fits into the simple definition of a fractal given earlier.

Now that the Cantor set is proven to be a fractal, we need to look at what its fractal dimension is. This can be done in a variety of ways that include finding the similarity dimension or the box counting dimension. These are two of the most basic ways to find a fractal dimension of a fractal curve. To find the similarity dimension all one basically has to do is find a way to scale down the self-similar parts of the object. The box counting dimension is easily done using computers and will be looked into later. In Fractals and Chaos by Paul S. Addison, Addison derives the formula for the similarity dimension by first dividing Euclidean shapes into self-similar parts. This was first done by looking at a line of 1 unit length. This is then divided into N segments which needs to be multiplied by a scaling factor so that the equation is:

$$N\varepsilon=1$$

Addison further develops his formula to look at a two-dimensional object such as a square. The square is considered to have a unit area of 1 and is divided into N squares each with a scaling area of ε^2 so that the equation for this figure is:

$$N*\varepsilon^2=1$$

Look at the exponents of the scaling factors in each of the equations; each of the exponents are equal to the Euclidean dimension of the object which leads us to an equation for the similarity dimension:

$$N*\varepsilon^D=1$$

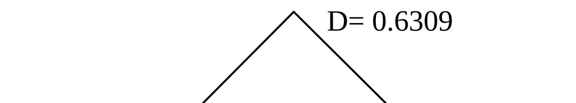
This through a little algebraic work can be solved for D:

D=

So with this new knowledge we can find what the similarity dimension of the Cantor set. Looking at the fourth line of Figure 1 we see two sets composed of two sets of two which are scaled down a third from the preceding line. So ~~N=2 and ε=1/3:~~

D=

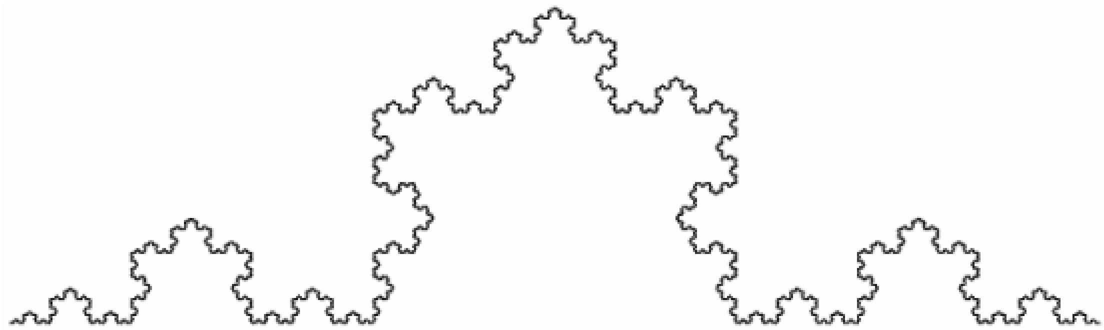
D= 0.6309



This means that Cantor set consists of something that is more than a point but is still something less than a line. A similar sort of thing occurs in the next most basic fractal, the Koch curve.

The Koch curve is another interesting fractal that can extend many of the major concepts that were introduced by the Cantor set. The Koch curve is created by taking a line of finite length and removing the middle third and replacing it with two equal segments, that are each one third in length. It is basically equivalent to removing the middle third of a line and replacing the piece with an two sides of an isosceles triangle. This looks like:

Eventually, after repeating this process an infinite number of times, this becomes:



With what is seen with this resolution four self-similar segments can be found, each which are one third the scale of the original line. So with N=4 and $\varepsilon=1/3$ we find that the similarity dimension is:

$$D=1.262$$

This means that the Koch curve falls somewhere in between a line and a two-dimensional Euclidean shape. This makes sense because the Koch curve has an infinite length but has no area, where a two dimensional shape has infinite length but also has area.

As mentioned previously, there is more than one fractal dimension. One of the most important fractal dimensions is considered to be the box dimension. This dimension is important because of how easy it is to use a computer to find the box dimension. To find the box counting dimension, we first cover an object with boxes, each with a certain side length. We then count how many boxes were required to cover the object and relate this to the side length of the cubes so that:

$$D_B= \log(N)/\log(1/\text{side length})$$

This is probably the most general way to figure out the estimated box dimension of an object. Computer programs do this in a slightly different way. The computer program covers the data with grids that have different box side lengths. Then the program counts the number of boxes that contain the data and then computes the box counting dimension using the formula:

$$D_B = [\log(N_2)-\log(N_1)] / [\log(1/\text{side length}_2) - \log(1/\text{sidelength}_1)]$$

This is concept will show it importance later when the course data sets are evaluated to find a fractal dimension of the data.

The final thing that one needs to consider when going over fractals is how these mathematical concepts can be applied to real world objects. Well one of the ways this can be applied is by looking around the natural world. It is not very often that natural objects both living and nonliving are found that can be described by pure Euclidean geometry. Consider a picture of lightning:

If one was to zoom in one any particular branch of the lightning's path it would be found that is looked similar enough to the lightning as a whole that we would find it hard to determine which the zoomed in image was and which the original picture of the lightning was. Addison calls these kinds of fractals, random fractals and describes them as being not exactly self-similar, like the Cantor set and Koch curve, but as statistically self-similar. This is why many things in nature can be described as random fractals. Here are just a few examples:

The individual leaves on the fren look like smaller versions of the fern as a whole, the boundary between the cloud and the air around it as well as the outline of the peaks of the mountain range can all be described by fractal mathematics because they are random fractals.

Benoit Mandelbrot, was the first person to mathematically describe all of these natural things as fractals but in recent years many scientists have been applying fractals to many different things and have discovered a lot about the patterns of the natural world. Some of the most intriguing uses of fractals have come from medical researchers modeling biological developments. In research published in 2008 by Mancardi, Varetto, Bucci, Maniero and Guiot, they showed that because fractals could be used to describe the complex network of the vascular system, fractals and the fractal dimension are a promising tool for monitoring the effectiveness of certain drugs (1). Another group has done fantastic research in the area of fractals is the research group led by K.J. Ray Liu who used fractals to model breast tissue so that they could remove noise from digital mammograms to better predict the presence of micro calcifications, which are sometimes a precursor to breast cancer (2). Other applications of fractals have included an analysis of the human heartbeat and how a healthy heart has a fractal waveform. Others have used fractals to study the proportions of one tree to predict how trees chosen at random from a forest have grown and then applied this to how much carbon dioxide is used by the forest during the photosynthesis cycle.

These have been just some of the various applications of fractals to science but have not included the application of fractals to meteorology. One of the first scientists to theorize that fractals could be applied to meteorology was Edward Lorenz. Lorenz is best known for his work on dynamic meteorology and on chaos. In his article “The Evolution of Dynamic Meteorology”, Lorenz reflects on how dynamic meteorology has developed over the past century and how his inclusion of chaos in the atmosphere has changed how meteorologists think about the atmosphere. He wrote “a chaotic system is one that exhibits sensitive dependence on initial conditions.” If there is a small change of any part of the system at any time it will lead to a completely differently state than would have otherwise occurred (4). His work with chaos lead him to the discovery of the Lorenz attractor shown below:

The attractor was derived from a simplified model of convection and shows how the atmosphere is chaotic and will never converge but will always diverge away from an equilibrium (3). The discovery of Lorenz and his ideas was the first development that lead to the idea of fractals being able to pull more fine data out of course data sets in meteorology.

However, Lorenz has not been the only scientist to apply the ideas of fractals to the atmosphere. In 2004, fractals were used to do an analysis on Indian climatic dynamics, such as the monsoon. They found that because the atmosphere was a fractal and chaotic system that it was hard to predict a monthly rainfall but with their fractal analysis they were able to predict a seasonal rainfall (5). This data showed me that fractals could be used to describe the weather patterns and it is because of these patterns in nature that makes one believe that fractals and fractal analysis techniques can be used to describe and make better sense of weather data collected by weather stations and observers.

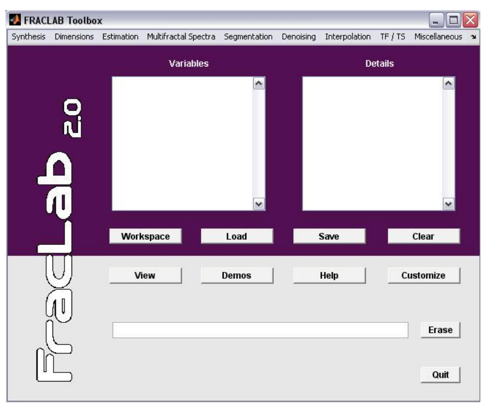
Methods and Experimentation

I. Data Sets Used

To compare the outcome of the fractal analysis of the course data, data needed to be found where both low resolution and high resolution data were present. The most accessible and some of the most useful data sets available come from weather radar centers. Data used in this experiment comes from the National Oceanic and Atmospheric Administration's National Weather Service (NWS). High Resolution radar comes from the national composite of base reflectivity radar that is compiled from all of the different radar towers and combined together to give high resolution (fine) data. To obtain the low resolution data, the NWS provides free radar information that is updated every six minutes that contains lower resolution (course) data that is combined together to form the high resolution data. This was obtained off of the NOAA's NWS website at weather.gov.

II. Computer Program

The program used in this project to assess the data sets is FracLab. It is “a general purpose signal and image processing toolbox bases on fractal and multi-fractal methods. FracLab is a free software that was developed by the Regularity team at Inria Saclay/Ecole Centrale de Paris” (6). The program runs on Matlab and can also be used as a standalone version that can run without the actual Matlab program being present. FracLab can perform many different tasks such as estimating the box dimension of a grayscale image, multi-fractal analysis of data, fractal denoising, interpolation and hundreds of other functions. The main uses of FracLab in this project were to estimate the box-dimension of radar data sets and to use the multi-fractal regularization (pseudo Kullback norm) function to find a fractal pattern in the data and enhance it. Multi-fractal analysis has been used a lot in recent years and has been able to provide useful tools in the development of Synthetic Apparatus Radar(SAR) data, this has been shown in various papers that also used the FracLab program. Below is the main screen of the FracLab program:



The version of FracLab used for this project was FracLab 2.05 Beta Stand-Alone.

III. Box Dimension Estimation

The first step in assessing the quality of the FracLab program was to use its box-dimension estimation feature on images and fractals that had an already known box dimension. This was done using a series of images created on Microsoft’s Paint program. The images are provided at the end of the paper with a description of what they are. These images were analyzed twice with differing parameters to see what parameters were going to be the most accurate for the analysis of the radar images. The following is what parameters were used:

Parameters 1=
Box Size
- Max Size = ½
- Min Size = 1/512
- Number of Box Sizes = 9
- Progression = Power Law
- Aspect Ratio 1:1:1
- Normalize Data

Regression
- Type = Least Square
- Range = Automatic

Parameters 2=
Box Size
- Max Size=1/128
- Min Size = 1/512
- Number of Box Sizes = 100

The rest of the parameters were kept the same as Parameters 1.

Each of the test images were put through the estimate box dimension function with each of the parameters. The following table summarizes the data collected from this experiment:

File Analyzed	Parameters 1 Estimated Box Dimension	Parameters 2 Estimated Box Dimension
Line Segment	1.8295	1.9842
Horizontal Line Segment	1.8639	1.9937
Rectangle	1.8194	1.9796
Circle	1.8084	1.9752
Two Circles	1.8046	1.9630
Four Circles	1.805	1.9516
Five Circles	1.7976	1.9412
Seven Circles	1.7915	1.9162

This showed that parameters like Parameters 2 would give a higher quality box dimension estimate than Parameters 1. After this the parameters were changed again so that there were a larger number of boxes or that the max box size was smaller, but each of these changes resulted in results that were similar or equal to those given by Parameters but they took two to four times longer to get the results. When testing the Seven Circles file at max box size=1/64 and the number of box sizes = 300, the estimated box dimension was 1.9172. This is only a 0.05% difference but took five minutes to complete the box dimension estimate compared to the thirty seconds it took to get the results of with Parameters 2. Each of these images should have obtained a box dimension of two. So they were well within a reasonable amount of error. This proved that the box dimension estimation feature could provide reliable data.

The other test used to prove the quality of FracLab's box dimension estimate was the testing of Koch's curve's box dimension. As shown earlier the fractal dimension of Koch's curve is 1.262. Using the same parameters as in Parameters 2, FracLab came up with a box dimension of 1.27. This is only a 0.6% difference and is within a reasonable amount of error. Because of the outcome of these tests later estimates of the box dimension of the radar images were considered reliable.

The next step was to determine if the radar data had a fractal dimension or if it was two dimensional data. So the same procedure was done as before only this time only Parameters 2 were used. Here are the results for this portion of the experiment:

File Name	Estimated Box Dimension
MLB_NOR_0.gif	1.7465
GRB_NOR_0.gif	1.7384
EPZ_NOR_0.gif	1.8899
JAX_NOR_0.gif	1.6425
JAX_NOV_0.gif	1.7955
JAX_NIP_0.gif	1.8812

This data shows that the radar images contain a certain amount of "fractalness". This was a great first step in determining if fractals could be used to help bring out more data. The fact that there was a fractal dimension present in the data shows that there could be an underlying pattern present in the data that could manipulated to help bring out a finer data sample. Fractal analysis could have been done on images that were not fractals, but the discovery that this data was a fractal helped solidify the fact that natural phenomena have a certain pattern that can be described using fractals.

All data file names with _NOR are base reflectivity radar images, _NOV is a velocity radar image, and _NIP is a rainfall radar image. MLB is radar from Melbourne, Florida. CRB is radar from Green Bay, Wisconsin. EPZ is from El Paso Texas. Each of these data sets were obtained from the National Weather Service on December 12, 2011, around approximately 1500 CST. All data labeled JAX is from Jacksonville, Florida and was obtained on the same day only at 0940 CST. The Jacksonville data was used as a test to see if the multi-fractal regularization could be used to enhance the radar data. It was not used for the fine resolution comparison. The actual data images will be provided later, next to the fractal representation of the data and the analyzed data.

IV. Multi-Fractal Regularization

The next step of the process was more of a trial and error step. Multiple attempts were made using different fractal image processing techniques. These included fractal denoising, fractal pumping, and Hölder segmentation. Each of these processes did not change the output of the data. Denoising and pumping smoothed over the data but did not pull out any new information. Hölder segmentation did not change the data in any observable way.

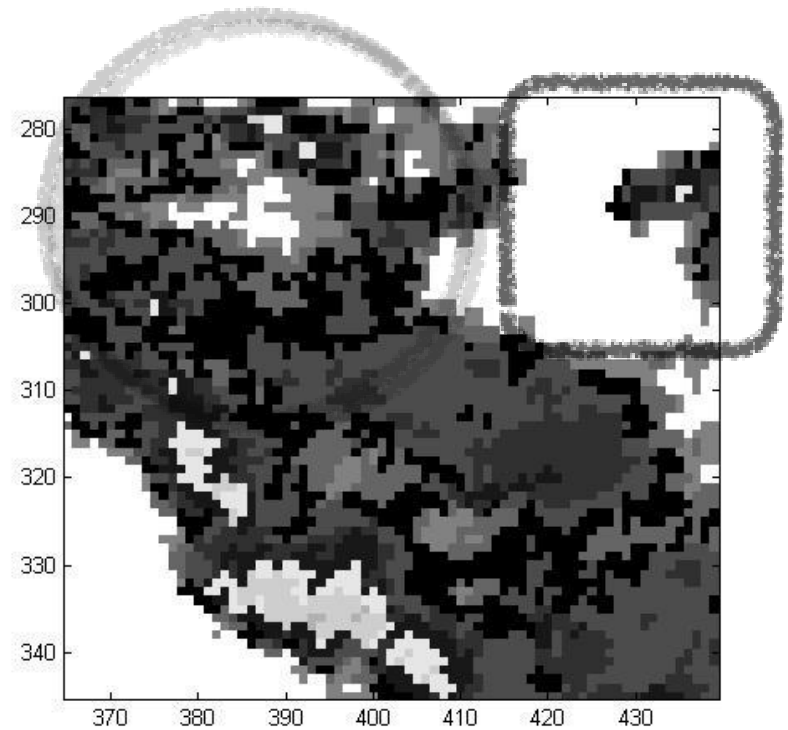
Multi-Fractal Regularization (pseudo Kullback norm) function on FracLab was the function that provided the best results. The basic way multi-fractal regularization works is that it is assumed that there is no noise in the image and then based on the fractal nature of the data the Hölder regularity is increased based on a wavelet (7). The Hölder regularity is determined by the Hölder exponent which is a way to describe the regularity of every point in the image (For a more detailed discussion of this topic see the paper by Dangeti on the Works Cited section).

While preforming the multi-fractal regularization three different parameters were used to see how each of them affected the data. This table summarizes the parameters used to analyze the radar data:

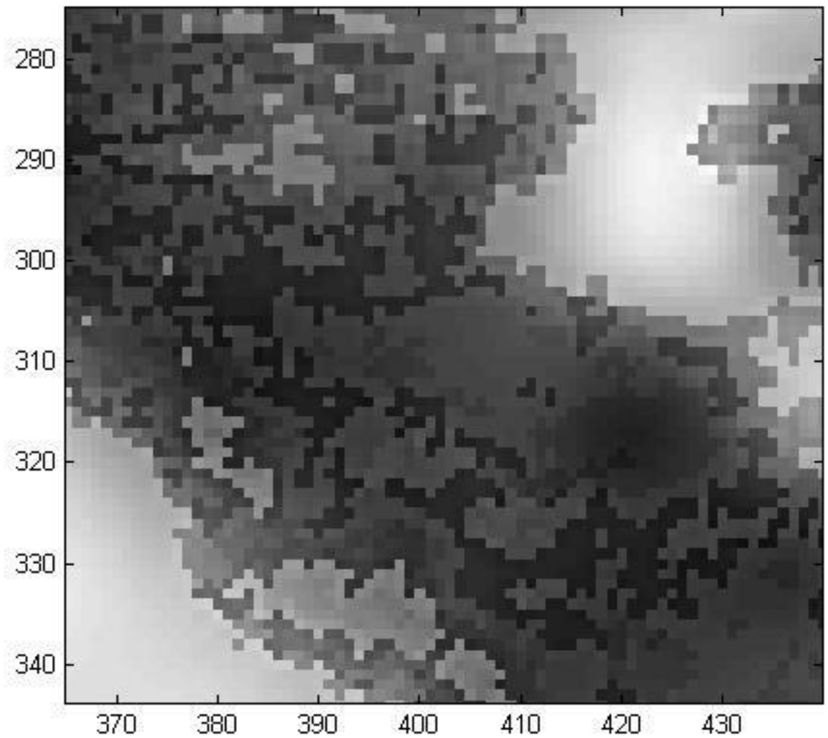
	Parameter 1 (P1)	Parameter 2 (P2)	Parameter 3 (P3)
Regularity Increase	1	1	1.2
Start Level	5	5	5
	Daubechies 10	Daubechies 18	Daubechies 10

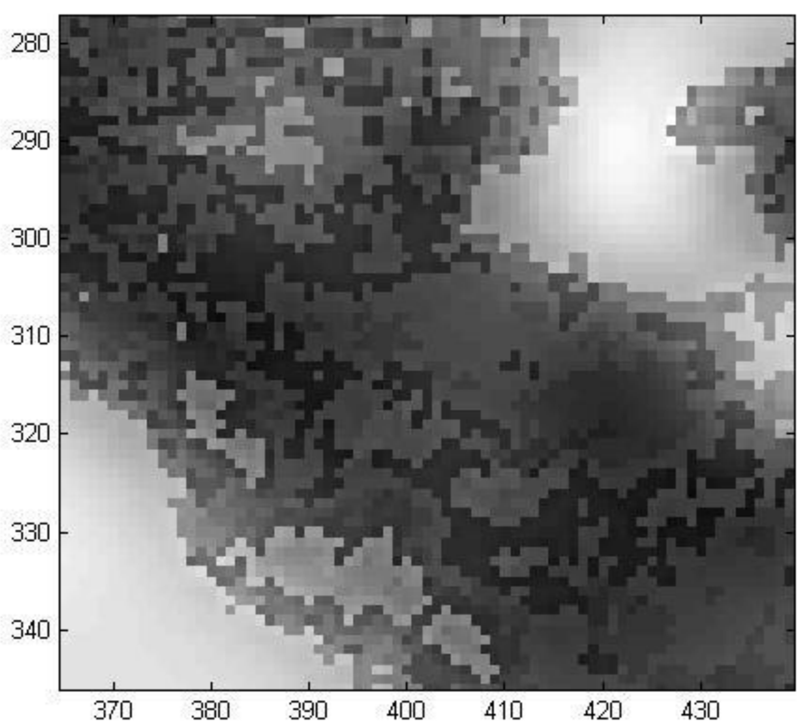
Each data sample was put through each of these parameters and looked fairly similar from a full view but as the analyzed data was zoomed in on some differences became apparent. All of the images are available (in large format) at the end of the paper after the Works Cited page and after the box dimensions test images. (ask Dave if I can give him digital copies of the images)

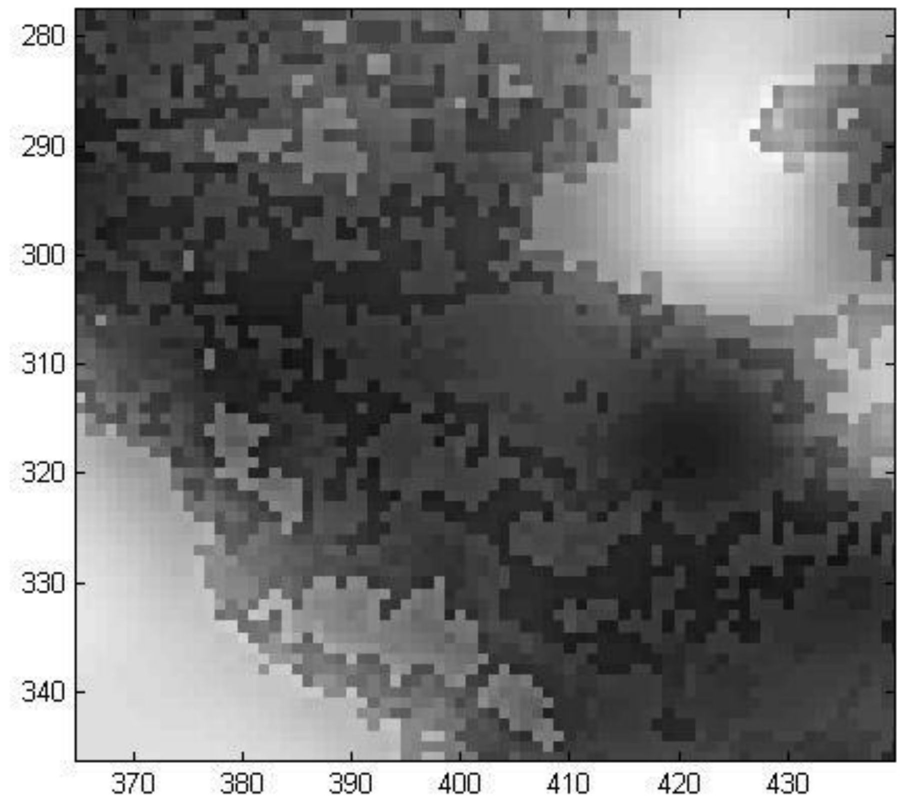
After using the multi-fractal regularization on the course data there were three new images to use to compare the data to the fine data set. These four images are from the Melbourne, Florida data and are zoomed in images of one of the cloud formations that is present in both the course and fine data sets:



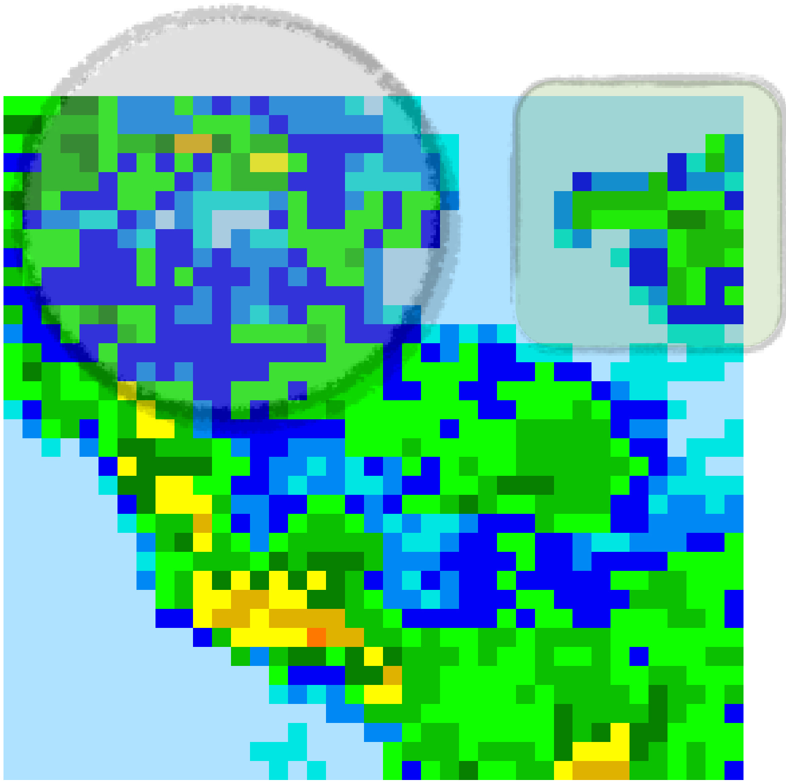
This is the course data radar sample. The two areas that will be focused on are in the circle and the rectangle. Notice in the circle the, large open patch of white space. Now comparing that to the same location in each of the following pictures:


280
290
300
310
320
330
340
370
380
390
400
410
420
430


280
290
300
310
320
330
340
370
380
390
400
410
420
430



Each of the fractal analyzed pictures had that area filled in to a greater extent than the course data sample did. If we compare that to the fine data sample set below we see that the fractal regularlized data is much closer to the fine data than to the course data sample.



As can be seen in the fine data set this blank spot is not as big as the course data would lead a forecaster to believe.

For reference the second picture above is the regularization data using paramters P1. The third picture is the the regularized data using parameters P2. The fourth picture is the regularized data using parameters P3.

As a more complete comparison of the analyzed data to the unanalyzed course data, it is seen that main of the main features are seen in each data set. Anything that is of dark color on the unanalyzed image is still dark in color compared to the analyzed image. But, if one looks at the analyzed data closely, it is easily seen that there are slight irregularities in the data as compared to the course data.

For another example of an increased amount of fine data look at the areas marked in the course data image and the fine data image that is in the rectangle. In the course data sample, the area is very basic and seems to consist of a sample pattern, but in the fine data sample there is actually quite a variety of data in that area. Comparing each of the analyzed data to the course and fine data, again it is seen that the analyzed data is more in line with the complexities present in the fine data sample.

Each of the data sets analyzed show that by using the fractal analysis the data becomes more like the fine data set. The only drawback that is present in the current state of the data is the lack of using FracLab to apply the correct color scale to the analyzed data. This would allow for a better judgement of the quality of the fractal analysis of the course data.

V. Conclusion

As has been shown throughout this paper, fractals play a key role in how scientists can make better sense of the nature. Over the past forty years fractals have risen from a place of obscurity in scietific communities and now are being used to make many wonderful advances in how dynamic systems are modeled, how physicians treat and diagnose patients, how engineers design new and highly advanced technological devices. However there is still much more to learn about fractals and how nature uses these patterns. Using fractals and fractal analysis techniques course weather radar data has been improved so that it is more comparable to the fine weather radar data. There are still imporvements to be made in how fractals can be applied to meteorological studies but hopefully this research has made a step in the right direction, so that more communities around the world will have better access to high-resolution, fine weather radar to be used to forecast or to warn people about dangerous weather phenomena.

http://principiacosmologica.blogspot.com/2010/07/chaos-theory-part-2.html (look at later)

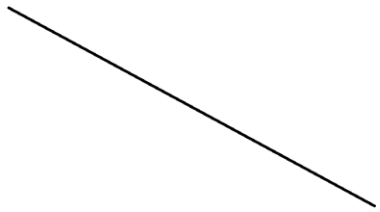
Works Cited

(2)Li, H., K.J.R. Liu, and S.-C.B. Lo. "Fractal Modeling and Segmentation for the Enhancement of Microcalcifications in Digital Mammograms." *IEEE Transactions on Medical Imaging* 16.6 (1997): 785-98. *UK PubMed Central*. 1997. Web. 15 Oct. 2011.

(1)Mancardi, Daniele, Gianfranco Varetto, Enrico Bucci, Fabrizio Maniero, and Caterina Guiot. "Fractal Parameters and Vascular Networks: Facts & Artifacts." *Theoretical Biology and Medical Modelling* 5.1 (2008): 12. *TBioMed*. 17 July 2008. Web. 15 Oct. 2011.

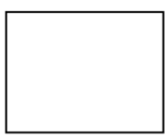
(3) http://paulbourke.net/fractals/lorenz/ (need to cite still.)
(4) Historical Essays on Meteorology (book need to cite still)
(5)FracLab website

7 http://etd.lsu.edu/docs/available/etd-1219102-152426/unrestricted/Dangeti_thesis.pdf need to cite

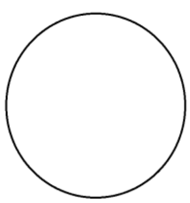


Line segment

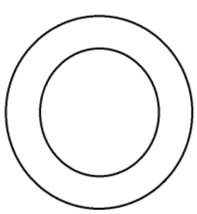
Horizontal Line segment



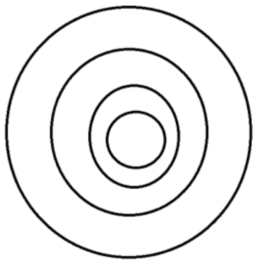
Rectangle



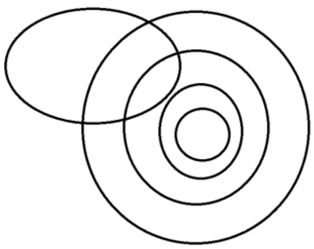
Circle



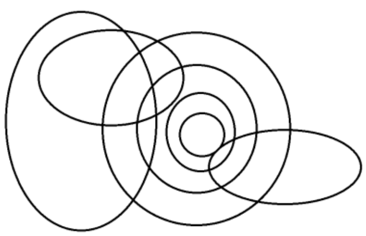
Two Circles



Four Circles



Five Circles



Seven Circles

Melbourne, Florida (approximately 1500 CST on 12/1211) Images